3-SPIN CORRELATIONS

IN ELECTRON COMPTON SCATTERING

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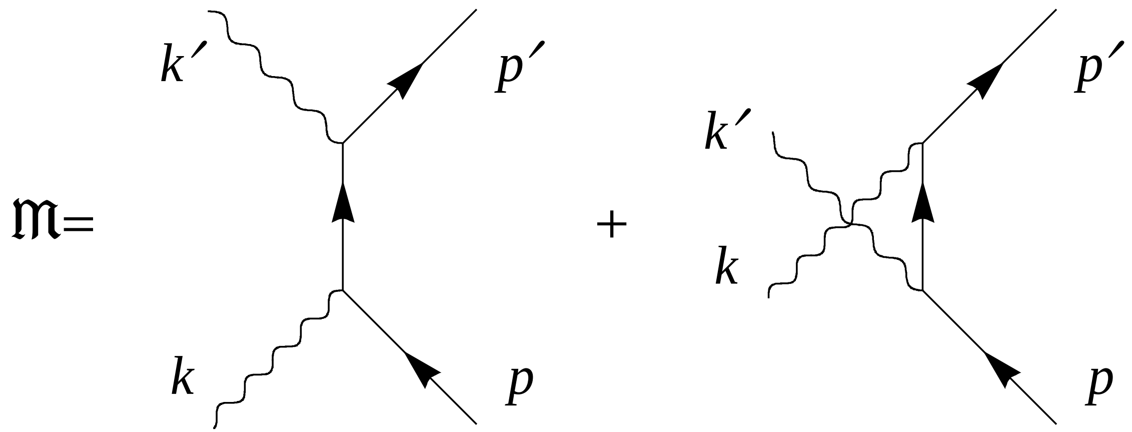
Outlook:

- ***The matrix amplitude reduction***

- ***3-spin correlations and their physical applications***

- ***Differential cross-section with all particle polarizations***



Matrix amplitude calculation

$$p^2 = p'^2 = m^2, \quad k^2 = k'^2 = 0$$

$$\mathfrak{M} = \bar{u}' \left\{ \frac{e'^* \cdot \gamma (p \cdot \gamma + k \cdot \gamma + m) e \cdot \gamma}{2 p \cdot k} - \frac{e \cdot \gamma (p \cdot \gamma - k' \cdot \gamma + m) e'^* \cdot \gamma}{2 p \cdot k} \right\} u$$

The simplest *relativistic* dynamics

BUT

The spin correlations are not transparent

Two steps to simplify the matrix amplitude (without resorting to any approximations):

1). <u>γ-matrix factorization</u>

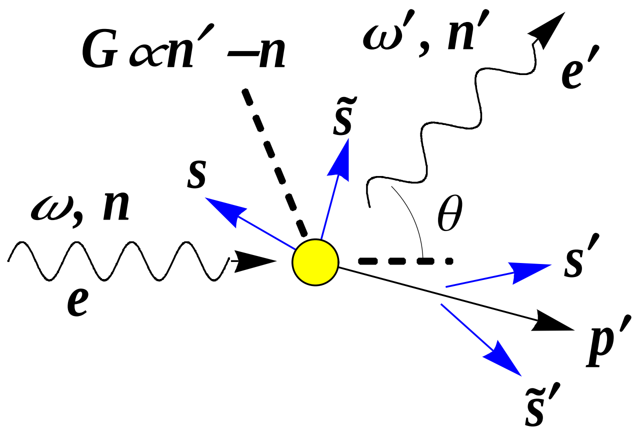
$$e_p \cdot p \overset{\text{def}}{=} 0 = e_p \cdot k, \qquad e'_p \cdot p \overset{\text{def}}{=} 0 = e'_p \cdot k'$$

$$\mathfrak{M} = \bar{u}' \left\{ e_p \cdot e'^*_p \frac{2 p \cdot k'}{p \cdot k - p \cdot k'} + e_p \cdot \gamma e'^*_p \cdot \gamma \right\} G \cdot \gamma u,$$

with $$G^\mu = \frac{k^\mu}{2 p \cdot k} - \frac{k'^\mu}{2 p \cdot k'}$$

2). Passage to reflected spinors

Go to the initial electron rest frame



$$p = (m, \mathbf{0})$$

$$u = \begin{pmatrix} w \\ 0 \end{pmatrix}$$ (in Dirac representation)

$$e_p = (0, \mathbf{e}), \quad e'^*_p = (0, \mathbf{e}'^*)$$

$$p \cdot k = m\omega, \quad p \cdot k' = m\omega'$$

$$\mathfrak{M} = \frac{1}{\sqrt{2m(E'+m)}} w'^\dagger \mathbf{p}' \cdot \boldsymbol{\sigma} \left\{ \mathbf{e} \cdot \mathbf{e}'^* \frac{\omega + \omega'}{\omega - \omega'} + i \left[\mathbf{e}, \mathbf{e}'^* \right] \cdot \boldsymbol{\sigma} \right\} (\mathbf{n} - \mathbf{n}') \cdot \boldsymbol{\sigma} w$$

$$\tilde{w} = \mathrm{R}_{\mathbf{n}-\mathbf{n}'} w, \qquad \mathrm{R}_{\mathbf{n}-\mathbf{n}'} = \frac{(\mathbf{n} - \mathbf{n}') \cdot \boldsymbol{\sigma}}{|\mathbf{n} - \mathbf{n}'|}, \qquad \tilde{w}^\dagger \tilde{w} = w^\dagger w$$

$$\tilde{w}' = \mathrm{R}_{\mathbf{p}'} w', \qquad \mathrm{R}_{\mathbf{p}'} = \frac{\mathbf{p}' \cdot \boldsymbol{\sigma}}{|\mathbf{p}'|}, \qquad \tilde{w}'^\dagger \tilde{w}' = w'^\dagger w'$$

$$\boxed{\mathfrak{M} = \tilde{w}'^\dagger \left\{ \mathbf{e} \cdot \mathbf{e}'^* \frac{\omega + \omega'}{\sqrt{\omega\omega'}} + i \left[\mathbf{e}, \mathbf{e}'^* \right] \cdot \boldsymbol{\sigma} \frac{\omega - \omega'}{\sqrt{\omega\omega'}} \right\} \tilde{w}}$$ - still exact but beneficially simple expression

3-spin correlations

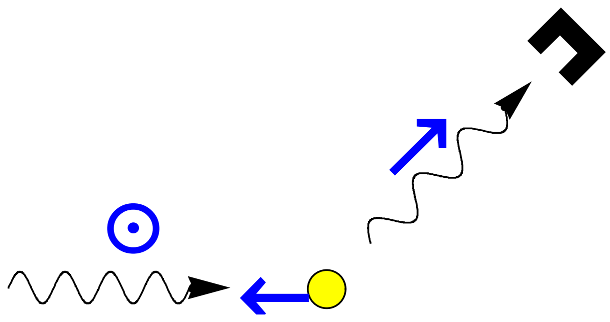
- ***Production of polarized high-energy photon beams from a polarized high-energy electron beam illuminated by a polarized laser;***
- ***monitoring electron polarization in a storage ring by measuring asymmetries of laser light scattered on it;***
- ***production of polarized positron beams in polarized $\gamma\gamma$ collisions***

Initial photon polarization linear, final photon circular

$\left[\boldsymbol{n}', \boldsymbol{e}'^{*}_{\lambda'}\right] = i\lambda' \boldsymbol{e}'^{*}_{\lambda'}, \qquad \lambda' = \pm 1$ - final photon helicity

1). $\boldsymbol{e} = \boldsymbol{e}_N \perp \boldsymbol{n}, \boldsymbol{n}'$ - initial photon linearly polarized transverse to the scattering plane

$$\mathfrak{M} = \frac{1}{\sqrt{2}} \tilde{w}'^{\dagger} \left\{ \frac{\omega + \omega'}{\sqrt{\omega\omega'}} + \lambda' \boldsymbol{n}' \cdot \boldsymbol{\sigma} \frac{\omega - \omega'}{\sqrt{\omega\omega'}} \right\} \tilde{w}$$



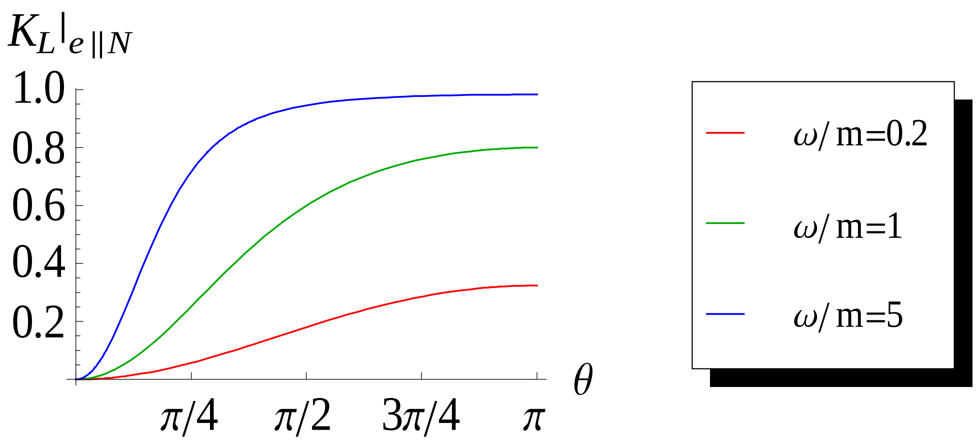
Yet do not forget to reflect the electron spin back:

$\mathrm{R}_{\boldsymbol{n}-\boldsymbol{n}'} \boldsymbol{n}' = -\boldsymbol{n}.$

Final photon - initial electron spin correlator (longitudinal only):

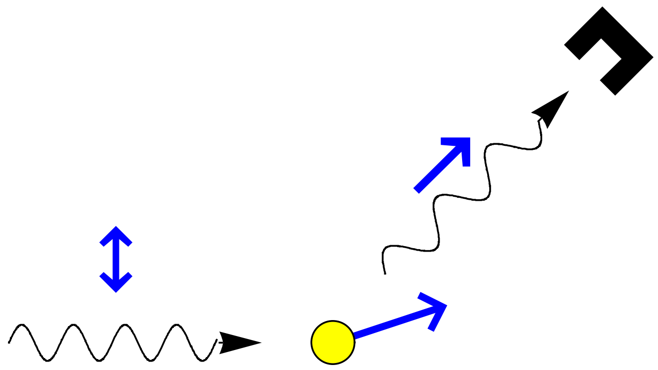
$$K_L\Big|_{\boldsymbol{e}\|\boldsymbol{N}} = \frac{\left|\mathfrak{M}(\lambda' = \sigma)\right|^2 - \left|\mathfrak{M}(\lambda' = -\sigma)\right|^2}{\left|\mathfrak{M}(\lambda' = \sigma)\right|^2 + \left|\mathfrak{M}(\lambda' = -\sigma)\right|^2}$$

$$= \frac{\omega^2 - \omega'^2}{\omega^2 + \omega'^2} = \frac{\left\{1 + (1 - \cos\theta)\,\omega/m\right\}^2 - 1}{\left\{1 + (1 - \cos\theta)\,\omega/m\right\}^2 + 1}$$



2). $\boldsymbol{e} = \boldsymbol{e}_S \parallel \left[\boldsymbol{n},\left[\boldsymbol{n},\boldsymbol{n}'\right]\right]$ - initial photon linearly polarized in the scattering plane

$$\mathfrak{M} = -\frac{i\lambda'}{\sqrt{2}}\tilde{w}'^{\dagger}\left\{\cos\theta\frac{\omega+\omega'}{\sqrt{\omega\omega'}} + \left(\lambda'\boldsymbol{n}' + i\left[\boldsymbol{n},\boldsymbol{n}'\right]\right)\cdot\boldsymbol{\sigma}\frac{\omega-\omega'}{\sqrt{\omega\omega'}}\right\}\tilde{w}$$

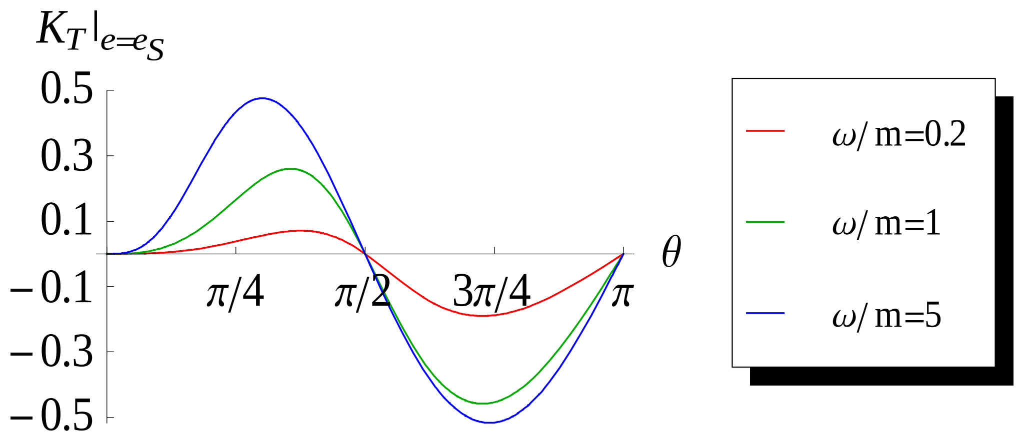


The amplitude is more complicated

BUT

There emerges a correlation with the <u>transverse</u> electron spin component:

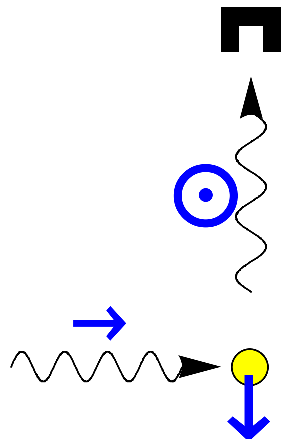
$$K_S\Big|_{\boldsymbol{e}_S} = \frac{\lambda'}{2}\sin 2\theta\frac{\omega^2-\omega'^2}{\omega^2+\omega'^2-\omega\omega'\sin^2\theta}$$



Initial photon polarization circular, final photon linear.

Scattering to 90° in initial electron rest frame

$$\left[\boldsymbol{e}'_\lambda, \boldsymbol{n}\right] = i\lambda \boldsymbol{e}'_\lambda, \qquad \lambda = \pm 1$$

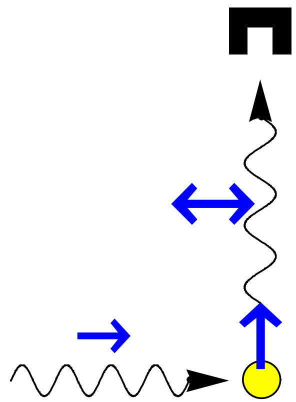


1). $\boldsymbol{e}' = \boldsymbol{e}'_N \perp \boldsymbol{n}, \boldsymbol{n}'$

$$\mathfrak{M} = \frac{1}{\sqrt{2}} \tilde{w}'^{\dagger} \left\{ \frac{\omega + \omega'}{\sqrt{\omega\omega'}} + \lambda \boldsymbol{n} \cdot \boldsymbol{\sigma} \frac{\omega - \omega'}{\sqrt{\omega\omega'}} \right\} \tilde{w}$$

$\mathrm{R}_{\boldsymbol{n}-\boldsymbol{n}'}\, \boldsymbol{n} = -\boldsymbol{n}'$ - access just to the transverse electron spin

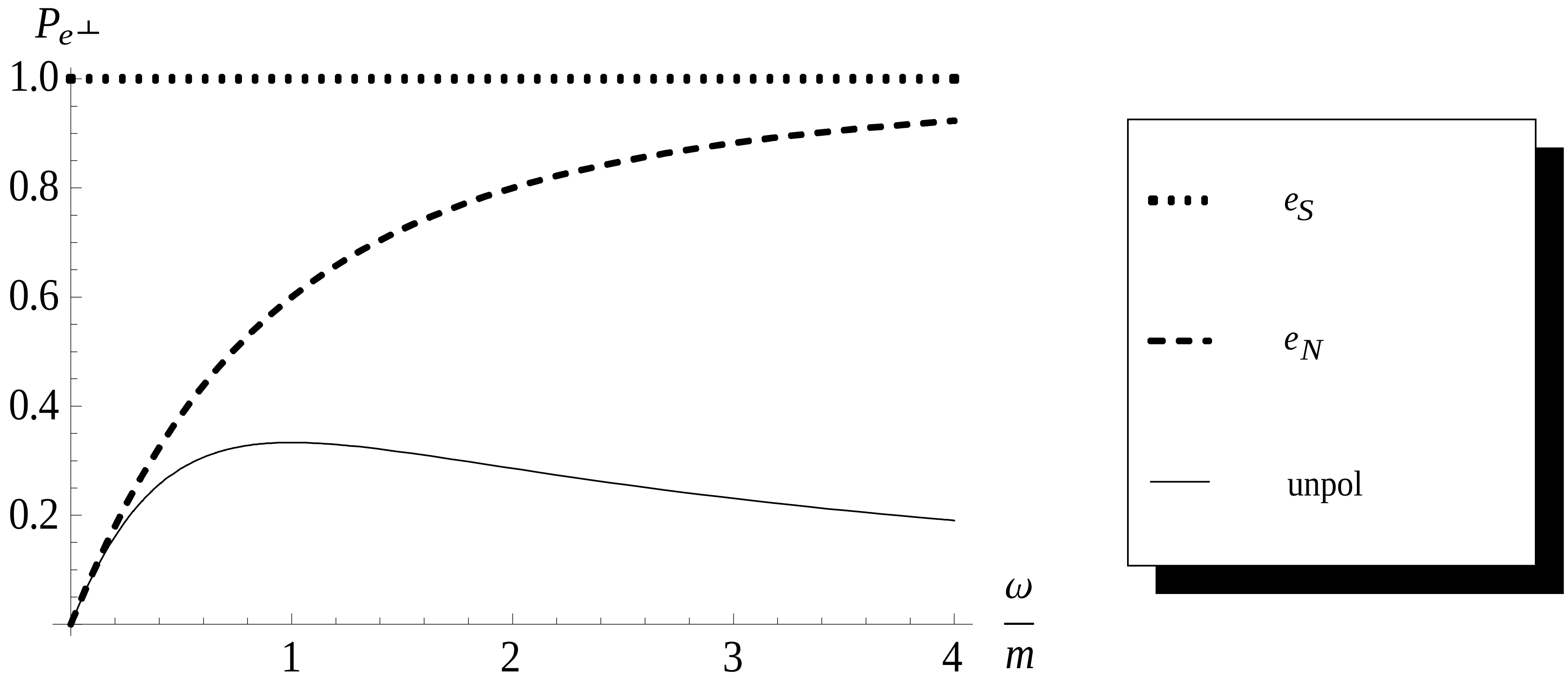
$$C_T\Big|_{\boldsymbol{e}' \| \boldsymbol{N}} = \frac{\left|\mathfrak{M}\left(\lambda = \sigma\right)\right|^2 - \left|\mathfrak{M}\left(\lambda = -\sigma\right)\right|^2}{\left|\mathfrak{M}\left(\lambda = \sigma\right)\right|^2 + \left|\mathfrak{M}\left(\lambda = -\sigma\right)\right|^2} = \frac{1}{1 + \dfrac{2m^2}{\omega\left(2m + \omega\right)}}$$



2). $\boldsymbol{e}' = \boldsymbol{e}'_S \parallel \boldsymbol{n} \perp \boldsymbol{n}'$

$$\mathfrak{M} = -\lambda \frac{\omega - \omega'}{\sqrt{\omega\omega'}} \tilde{w}'^{\dagger} \boldsymbol{e}_\lambda \cdot \boldsymbol{\sigma} \tilde{w} = \sqrt{2} \frac{\omega - \omega'}{\sqrt{\omega\omega'}} \delta_{\lambda -\sigma} \delta_{\lambda\sigma'}$$

$$C_T\Big|_{\boldsymbol{e}'_S} = \frac{\left|\mathfrak{M}\left(\lambda = \sigma\right)\right|^2 - \left|\mathfrak{M}\left(\lambda = -\sigma\right)\right|^2}{\left|\mathfrak{M}\left(\lambda = \sigma\right)\right|^2 + \left|\mathfrak{M}\left(\lambda = -\sigma\right)\right|^2} = 1$$ - perfect transverse spin filter!



$P_{e^\perp}$
1.0
0.8
0.6
0.4
0.2
1
2
3
4
$\frac{\omega}{m}$
e_S
e_N
unpol

Conclusions

- ***The amplitude of Compton scattering at arbitrary energy may be reduced to an amplitude of light scattering on a recoilless system with simple dispersion. The correspondence for electron spinors is provided by specific operators of reflection.***
- ***If the initial photon is polarized linearly transversely to the scattering plane, then the final photon helicity correlates only with the initial electron's longitudinal polarization.***

- ***If the initial photon is polarized linearly in the scattering plane, then the final photon helicity correlation with the initial electron's transverse polarization exists, but it does not exceed 50%.***
- ***If the photon is scattered to 90° in IERF and final photon is linearly polarized in the scattering plane, then the initial photon helicity correlates only with the initial electron's transverse polarization.***
- ***The abovementioned correlations involving electron transverse polarization may be utilized for high-efficiency polarimetry of electron beams in circular accelerators.***