HARD SCATTERING ON LIGHT NUCLEI: A CONVENIENT WAY TO STUDY PARTON CORRELATIONS

Giorgio Calucci and Daniele Treleani

Dipartimento di Fisica

dell'Università di Trieste

and INFN Sezione di Trieste - Italia

• • • • 2011 - АЛУШТА • • ••

MOTIVATIONS

The one-body partonic distributions in the hadrons are well investigated using electromagnetic or weak interactions.

If we wish to exploit the same procedure to study the two-body distributions we should study the very rare events with multiple electromagnetic or weak interactions on the same hadron.

The alternative is to study events with hard QCD double scattering of partons of the same hadron, such events become more and more abundant when the energy of the colliding hadrons grows. In fact at very high energies even the parton at small fractional momentum x may suffer collisions with momentum transfer large enough to allow a perturbative treatment.

We wish to show:

The way of performing the calculations with a sufficient general starting point.

An example where the calculations are led to more explicite results at the price of loosing generality.

Because we think that this kind of experiment, if performed, is a useful tool to investigate a sector of non perturbative QCD that is less explored that other aspects of QCD.

technical note: *the problem has three scales: the small scale of the hard interaction, the scale of the hadron, the scale of the nuclei.*

PARTIALLY QUANTITATIVE DESCRIPTION

Usual parametrizations: *Effective cross section:* $\sigma_{\text{eff}} = {\sigma_S}^2/(2 \cdot \sigma_D)$.

σ_S: integrated inclusive cross section for one hard scattering,

σ_D: integrated inclusive cross section for two hard scatterings.

Intuitively: $\sigma_{\text{eff}} \Leftrightarrow$ [size of the hadron].

True but not completely: possible effects from

correlations among the partons and differences in multiplicity distribution.

With limited integration: $x_o - \Delta x < x < x_o + \Delta x \;\; x'_o - \Delta x < x' < x'_o + \Delta x \rightarrow \sigma_{\text{eff}}\big|_{x_o x'_o}$

Two extreme situations:

1 Configurations with high multiplicity are frequent

$\rightarrow$ many double collisions $\rightarrow$ small σ_{eff}

2 Partons strictly correlated, if one collides, another collides $\rightarrow$ small σ_{eff}

We can go on further: for $K-$hard scatterings define the dimensionless parameters τ_K *through*

$$\sigma_K = \frac{(\sigma_S)^K}{K!(\sigma_{\text{eff}})^{K-1}\tau_K} \qquad \tau_2 = 1$$

In particular τ_3 *says someting on three-body correlations.*

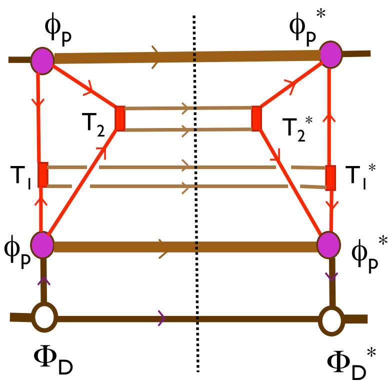
To start consider double scattering - possible cases:

1 - one bound nucleon interacts twice, the other is spectator

2 - both bound nucleons interact once

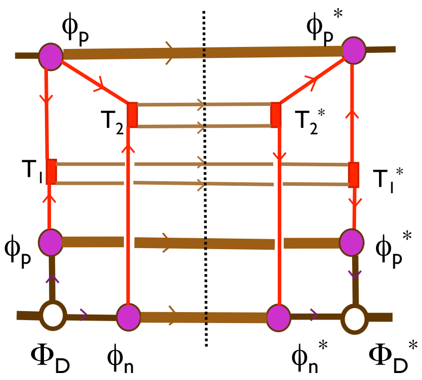
3 - both bound nucleons interact once, but produce a quantum interference term.

see graphs: next slide



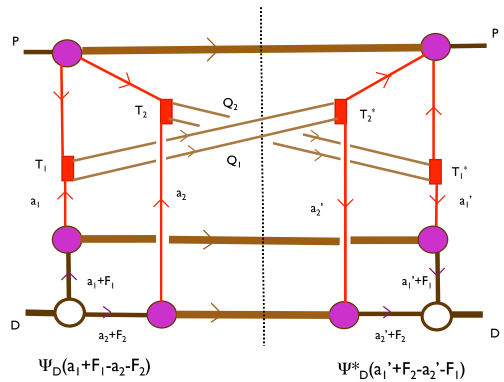
ϕ_p
ϕ_p^*
T_2
T_2^*
T_1
T_1^*
ϕ_p
ϕ_p^*
Φ_D
Φ_D^*

a)



ϕ_p
ϕ_p^*
T_2
T_2^*
T_1
T_1^*
ϕ_p
ϕ_p^*
Φ_D
ϕ_n
ϕ_n^*
Φ_D^*

b)



P
P
T_2
Q_2
T_2^*
T_1
Q_1
T_1^*
a_1
a_2
a_2'
a_1'
a_1+F_1
$a_1'+F_1$
D
D
a_2+F_2
$a_2'+F_2$
$\Psi_D(a_1+F_1-a_2-F_2)$
$\Psi^*_D(a_1'+F_2-a_2'-F_1)$

1 - As if the interacting nucleon were free, no new information.

2 - Effects of multiplicity distribution and possible transverse correlations different on proton and on the deuteron side.

3 - As in 2, but mismatch between *left and right* nuclear wave functon $\rightarrow$ smaller contribution. *''Crossed'' diagram: longitudinal momentum carried away at left is different from longitudinal momentum carried away at right; in transverse plain both interactions are localized in the same hard region.*

PROCEDURE and SYMBOLS

Formally: Feynman rules supplemented by:

effective vertices nucleon-1,2,3 partons $\phi, \widehat{\phi}, \breve{\phi}$

nuclear wave functions=nonrelativistic w.f.+boost Ψ

perturbative parton-parton scattering amplitude T

EXAMPLES:

Analytic expression when one nucleon interacts twice: graph 6a

$$\text{Disc}\mathcal{A} = \int dQ_{1,\pm}\, dQ_{2,\pm} \mathcal{W}_{2,0} \qquad Q_i = q_i + q_i'$$

Integration in $Q_{i,\pm}$ over all the hard scattering region.

$$\begin{aligned}
\mathcal{W}_{2,0} \;&= \frac{1}{(2\pi)^{21}} \int \frac{\widehat{\phi}_p}{l_1^2 l_2^2} \frac{\widehat{\phi}_p^*}{{l_1'}^2 {l_2'}^2} \frac{\widehat{\phi}_p}{a_1^2 a_2^2} \frac{\widehat{\phi}_p^*}{{a_1'}^2 {a_2'}^2} \chi(D,N)\chi^*(D,N') \\
\times &\quad T_1(l_2, a_2 \to p_1, p_2)\; T_1^*(l_2', a_2' \to p_1, p_2)\; T_2(l_1, a_1 \to q_1, q_2)\; T_2^*(l_1', a_1' \to q_1, q_2) \\
\times &\quad \delta(L - l_1 - l_2 - F_3)\; \delta(L - l_1' - l_2' - F_3)\; \delta(D - N - a_1 - a_2 - F_1)\; \delta(D - N' - a_1' - a_2' - F_1) \\
\times &\quad \delta(N - F_2)\delta(N' - F_2)\; \delta(l_1 + a_1 - Q_1)\; \delta(l_1' + a_1' - Q_1)\delta(l_2 + a_2 - Q_2)\; \delta(l_2' + a_2' - Q_1) \\
\times &\quad \prod_{i,j} d\Omega_i\; d^4a_i d^4a_i' d^4l_i d^4l_i'\; d^4N d^4N'\; d^4F_j \delta({F_j}^2 - {M_j}^2) dM_j^2 d^2Q_{i\perp}
\end{aligned}$$

Analytic expression when both nucleons interact once: graph 6b

$$\text{Disc}\mathcal{A} = \int dQ_{1,\pm}\, dQ_{2,\pm} \mathcal{W}_{1,1} \qquad Q_i = q_i + q_i'$$

Integration in $Q_{i,\pm}$ over all the hard scattering region.

$$\begin{aligned}
\mathcal{W}_{1,1} &= \frac{1}{(2\pi)^{21}} \int \frac{\widehat{\phi}_p}{l_1^2 l_2^2} \frac{\widehat{\phi}_p^*}{{l_1'}^2 {l_2'}^2} \frac{\phi_p}{a_1^2} \frac{\phi_p^*}{{a_1'}^2} \frac{\phi_n}{a_2^2} \frac{\phi_n^*}{{a_2'}^2} \chi(D;N)\chi^*(D;N') \\
&\times \; T_1(l_2, a_2 \to p_1, p_2)\, T_1^*(l_2', a_2' \to p_1, p_2)\, T_2(l_1, a_1 \to q_1, q_2)\, T_2^*(l_1', a_1' \to q_1, q_2) \\
&\times \; \delta(L - l_1 - l_2 - F_3)\, \delta(L - l_1' - l_2' - F_3)\, \delta(N - a_2 - F_2)\, \delta(N' - a_2' - F_2) \\
&\times \; \delta(D - N - a_1 - F_1)\, \delta(D - N' - a_1' - F_1)\, \delta(l_1 + a_1 - Q_1)\, \delta(l_1' + a_1' - Q_1) \\
&\times \; \delta(l_2 + a_2 - Q_2)\, \delta(l_2' + a_2' - Q_2) \\
&\times \; \prod_{i,j} d\Omega_i\; d^4a_i d^4a_i' d^4l_i d^4l_i'\; d^4N d^4N'\; d^4F_j \delta({F_j}^2 - {M_j}^2)\; d{M_j}^2 d^2Q_{i\perp}
\end{aligned}$$

the interference term has the same ingredients, but different $\delta-$functions.

General attitude in the following:

There are FORWARD particles, with large, $\propto \sqrt{s}$ $plus-$momentum and small $\propto 1/\sqrt{s}$ $minus-$momentum.

There are BACKWARD particles, with large, $\propto \sqrt{s}$ $minus-$momentum and small $\propto 1/\sqrt{s}$ $plus-$momentum.

There are particles where both components of the longitudinal variables can grow (they correspond to the produced particles).

The transverse variables are limited but their range does not depend on s. On the transverse components the *Fourier transformation* is performed, so we end with an expression in terms of fractional longitudinal momenta and transverse coordinates. The hard interaction is local in comparison with the hadron size.

When in a $\delta-$functions large and small components are present the small ones are neglected in the $\delta-$function and integrated separately.

This procedure is employed also in the bound-state wave functions.

THREE-BODY WAVE FUNCTION

The relative motion can be non relativistic, but there is a relativistic boost.

Follow Salpeter reduction of the 4dim. w.f. to 3dim. w.f.

Bethe-Salpeter equation:

$$\chi_D(q) = \Delta(N)\Delta(N')\int V(k)\chi_D(q-k)d^4k \qquad N = D/2+q,\ N' = D/2-q \tag{1}$$

for three bodies:

$$\begin{aligned}\chi_3(q_i) &= \Delta(q_1)\Delta(q_2)\sum_{J\neq 3}\int i\mathcal{T}_3(q_1+q_2,k)\chi_3(q_1+k,q_2-k,q_3)d^4k \\ \mathcal{T}_3 &= V_3 + G_3V_3\mathcal{T}_3\end{aligned} \tag{2}$$

Assumption: independence of the interaction and of the $\mathcal{T}$-matrix on the $time-$component of the relative momentum.

Ansatz: $\chi_J(q_1,q_2,q_3) = \Delta(q_1)\Delta(q_2)\Delta(q_3)\Phi_J(\mathbf{q_1},\mathbf{q_2},\mathbf{q_3})$,

consistent with the previous assumption.

Integrate on time component, neglect the antiparticle pole: If

$$\phi_3 = 2\pi G_o(E_B)\sum_{J\neq 3}\int \mathcal{T}_3\phi_J d^3k$$

$$G_o(E_B) = \frac{1}{E_B - \sum_i \kappa_i}$$

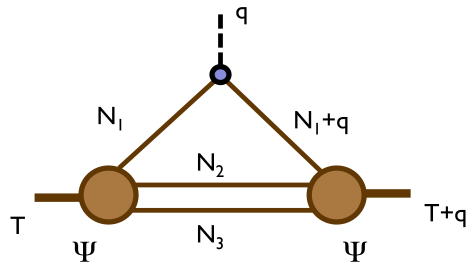
(normalized solution of Faddeev equation), then $\phi_3 = \mathcal{N} G_o \Phi_3$.

NORMALIZATION

Relativisic normalization through charge conservation.

The limit $q \to 0$ gives the total charge

Asking the bound state to have the correct charge fixes $|\Psi|^2$



An example of the elaboration: from $\mathcal{W}_{1,1}$ we get

$$\begin{aligned}\sigma_{1,1} &= \frac{1}{(2\pi)^3}\int \Gamma(x_1,x_2,b_1,b_2)\Gamma(x'_1/Z,\beta_1)\Gamma(x'_2/(2-Z),\beta_2)\\ &\times \frac{d\hat\sigma(x_1,x'_1)}{d\Omega_1}\frac{d\hat\sigma(x_2,x'_2)}{d\Omega_2}\,|\Psi_D(Z,B)|^2\\ &\times dBdZ\prod_{i=1,2} db_i d\beta_i dx_i dx'_i d\Omega_i \delta(B+b_1-b_2-\beta_1+\beta_2)\end{aligned}$$

The factors are obtained from the previous expression: *One-body factors*

$$\psi(z,a_\perp|M^2)=\phi/a^2 \qquad a^2\equiv a_+a_- - {a_\perp}^2$$

$$\tilde\psi(z,\beta|M^2)=(2\pi)^{-1}\int\psi(z,a_\perp|M^2)\,\mathrm{exp}[-ia_\perp\beta]da_\perp$$

$$\Gamma(z,\beta)=\frac{1}{2(2\pi)^3}\int\left|\tilde\psi(z,\beta|M^2)\right|^2\frac{z}{1-z}\,dM^2$$

Two-body factors

$$\psi(z_1, z_2; a_{1\perp}, a_{2\perp}|M^2) = \frac{\phi}{a_1^2 a_2^2} d\alpha_+ \qquad \alpha = (a_1 - a_2)/2$$

$$\tilde{\psi}(z_1, z_2; \beta_1, \beta_2|M^2) = \frac{1}{(2\pi)^2} \int \psi(z_1, z_2; a_{1\perp}, a_{2\perp}|M^2) \exp[-ia_{1\perp}\beta_1 - ia_{2\perp}\beta_2] \prod da_\perp$$

$$\Gamma(z_1, z_2; \beta_1, \beta_2) = \frac{1}{4(2\pi)^6} \int \left|\tilde{\psi}(z_1, z_2; \beta_1, \beta_2|M^2)\right|^2 \frac{2z_1 z_2}{1 - z_1 - z_2} N_-^2 \, dM^2$$

In the same way: $\sigma_{2,0}$ from $\mathcal{W}_{2,0}$,but with an important difference:

In $\sigma_{2,0}$ the partonic variables and the nuclear variables are not connected, the nuclear variables are integrated separately.

In $\sigma_{1,1}$ the partonic variables and the nuclear variables are connected.

Only $\sigma_{1,1}$ may give someting new (with respect to nucleon-nucleon).

Now we consider the triple scattering:

in this case we consider also the TRITON because there is at least one process that is possible there and not on the DEUTERON.

Analitic expressions $\rightarrow$ Quite cumbersome.

Graphical presentation of three situations:

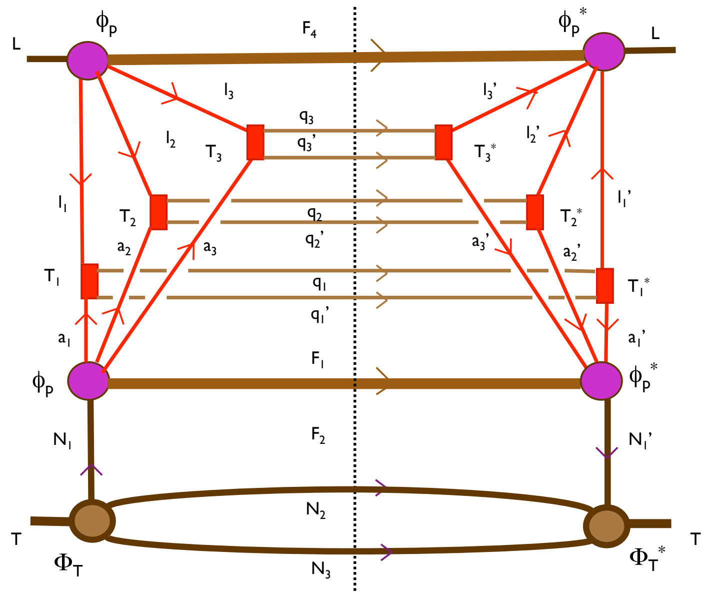
1 - one bound nucleon interacts thrice + one spectator (DEUTERON) or two spectators (TRITON).

2 - one bound nucleon interacts twice, another interacts once + one spectator (TRITON).

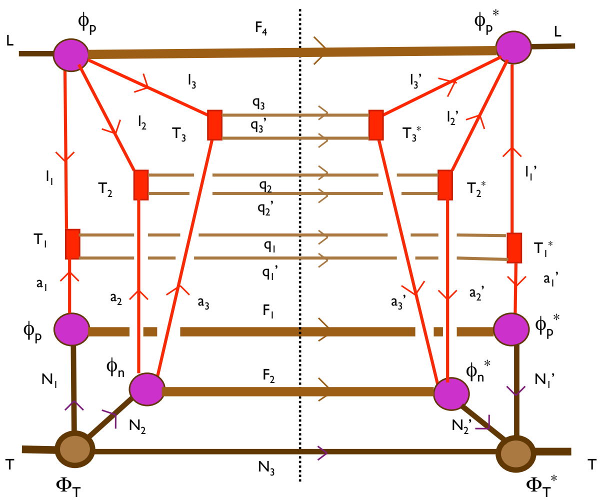
3 - three bound nucleons interact once (only TRITON).

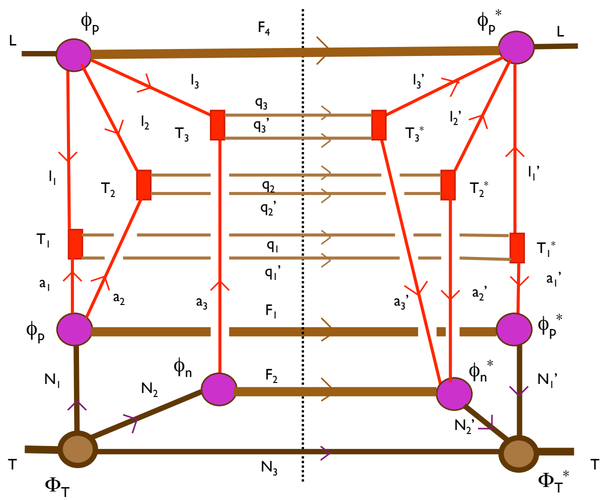
Crossed diagrams are not considered, they are subdominant as previously discussed.

1 This is the less interesting graph: the nuclear variables are not connected to the partonic varables. As if the nucleon were free, normalization a part.

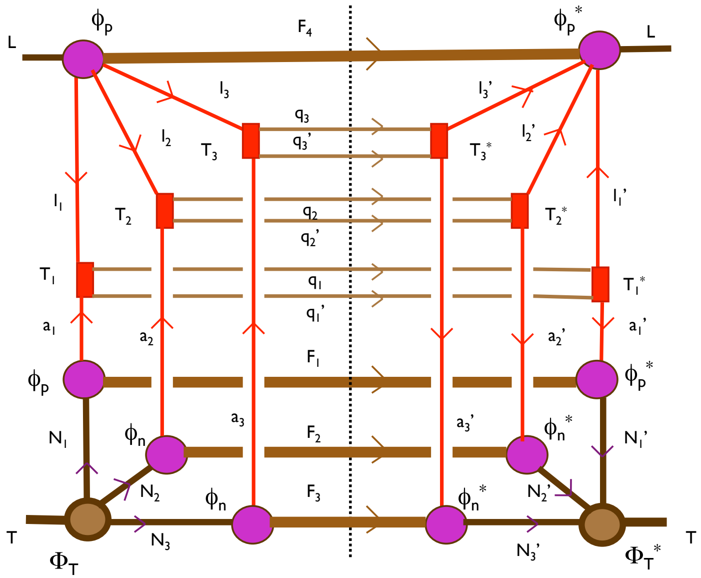


2 Two graphs :the second is a non diagonal term mismatch of the nuclear w.f., different longitudinal momenta similar to the crossed graphs. Only the first is considered.





3 This graph can refer only to TRITON



As an example: the expression corresponding to the last graph

$$\begin{aligned}\sigma_{1,1,1} &= \frac{3}{(2\pi)^3}\Gamma(x_1,x_2,x_3;s_1,s_2,s_3)\Gamma(\bar{x}_1/Z_1,b_1)\Gamma(\bar{x}_2/Z_2,b_2)\Gamma(\bar{x}_3/Z_3,b_3)\\ &\times \frac{d\widehat{\sigma}}{d\Omega_1}\frac{d\widehat{\sigma}}{d\Omega_2}\frac{d\widehat{\sigma}}{d\Omega_3}\left|\Psi\right|^2\delta(Z_1+Z_2+Z_3-3)dZ_1dZ_2dZ_3\\ &\times \delta(b_1-b_2-s_1+s_2-B_1+B_2)\,\delta(b_1-b_3-s_1+s_3-B_1+B_3)\\ &\times d(B_1-B_2)d(B_1-B_3)\prod_i ds_i\,db_i\,dx_i\,d\bar{x}_i\,d\Omega_i\ .\end{aligned}$$

$$\Gamma(x_1,x_2,x_3;s_1,s_2,s_3)=\frac{1}{2(2\pi)^9}\int|\tilde{\psi}(x_1,x_2,x_3;s_1,s_2,s_3)|^2\frac{x_1x_2x_3}{1-x_1-x_2-x_3}{L_+}^4dM_4^2$$

$$\psi_3(l_{1\perp},l_{2\perp},l_{3\perp})=\breve{\phi}_p\cdot\frac{(2\pi)^2}{{L_+}^2}\frac{1}{x_1x_2l_{3\perp}^2+x_2x_3l_{1\perp}^2+x_3x_1l_{2\perp}^2+[M_\perp^2/(1-\sum x_i)-m^2]x_1x_2x_3}\ .$$

A much simplified model:

Only the transverse degrees of freedom are treated explicitly with Gaussian distributions, the longitudinal degrees of freedom are in principle observables: so they are kept fixed, sometimes, at the end, they are integrated.

One-parton inclusive distribution:

$$\Gamma_1 = G(x)D_1(b) \quad D_1(b) = \frac{1}{\pi R^2}\,\mathrm{exp}[-b^2/R^2]$$

The two-parton distribution:

$$\Gamma_2 = G(x_1)G(x_2)K_2D_2(b_1,b_2) \quad D_2(b_1,b_2) = \frac{1}{(\pi R^2)^2(1-\lambda^2)}\,\mathrm{exp}[-(b_1^2+b_2^2+2\lambda b_1 b_2)/R^2(1-\lambda^2)]$$

Three-parton distribution:

$$\begin{aligned} \Gamma_3 &= G(x_1)G(x_2)G(x_3)K_3D_3(b_1,b_2,b_3) \\ D_3(b_1,b_2,b_3) &= \frac{1}{(\pi R^2)^3(1-2\lambda)(1+\lambda)^2}\,\mathrm{exp}\Big[-\frac{(1-\lambda)\sum_i b_i^2 + 2\lambda\sum_{i<j}(b_i\cdot b_j)}{R^2(1+\lambda)(1-2\lambda)}\Big] \end{aligned}$$

It results:

$$\int D_2(b_1, b_2) db_2 = D_1(b_1) \qquad \int D_3(b_1, b_2, b_3) db_3 = D_2(b_1, b_2) \; .$$

K controls the parton multiplicities, λ the spatial correlations.

If the distribution integrated on the longitudinal variables were Poissonian, the all the K_N would be 1.

Even a perfect Poissonian distribution of the partons' multiplicities can support spatial corralations if $\lambda \neq 0$.

Nuclear distribution, *i.e.* the square of the wave function

Deuteron:

$$f(Z,B) = W(Z)F(B) \quad F(B) = \frac{1}{\pi S^2}\exp[-B^2/S^2] \qquad 0 < Z < 2 \quad B = B_1 - B_2$$

Triton:

$$\begin{aligned} f(Z_i,B_i) &= W(Z_1,Z_2,Z_3)\delta(Z_1+Z_2+Z_3)F(B_i) \qquad 0 < Z < 3 \\ F(B_i) &= \frac{4}{3(\pi S^2)^2}\exp\left[-2[(B_1-B_2)^2+(B_2-B_3)^2+(B_2-B_3)^2]/3S^2\right] \end{aligned}$$

The normalization is

$$\int W(Z)dZ = 1, \quad \int W(Z_1,Z_2,Z_3)\delta(Z_1+Z_2+Z_3)dZ_1dZ_2dZ_3 = 1$$

Given the non relativistic dynamics $Z \approx 1$.

Simple double and triple hard scattering among free nucleons. The simple hard scattering

$$\sigma_1 = \int \hat{\sigma}_{xx'} G(x) G(x') dx dx'$$

The double hard scattering

$$\sigma_2 = \left[\int \hat{\sigma}_{xx'} G(x) G(x') dx dx' \right]^2 \frac{1}{4\pi R^2} \frac{K_2^2}{1+\lambda}$$

The triple hard scattering

$$\sigma_3 = \left[\int \hat{\sigma}_{xx'} G(x) G(x') dx dx' \right]^3 \frac{1}{(4\pi R^2)^2} \frac{K_3^2}{1+\lambda}$$

When only one bound nucleon interacts nothing essentially different from the free case happens, at least for internal non relativistic motion.

New features appear when more than one bound nucleon interact.

$$\sigma_{D,2}(x_1, {x'}_1; x_2, {x'}_2) = K_2 \prod_{i=1,2} \hat{\sigma}(x_i, {x'}_i) \frac{G(x_1)G({x'}_1)G(x_2)G({x'}_2)}{\pi[S^2 + 2(2+\lambda)R^2]}$$

$$\sigma_{D,3}(x_1, {x'}_1 \ldots x_3, {x'}_3)\Big|_d = K_3 K_2 \prod_{i=1,3} [\hat{\sigma}(x_i, {x'}_i)G(x_i)G({x'}_i)]) \frac{(1-\lambda)^3(1+\lambda)^2}{(2\pi)^2 R^2[S^2\mu(\lambda) + R^2\nu(\lambda)]}$$

μ and ν are known polynomials in λ. The interference term could be calculated within the model, but the general argument that says that it is less important still holds.

Three target nucleons interact, in this case we have necessarily a triton and the expression of $\sigma_{T,3}$ is

$$\sigma_{T,3}(x_1, {x'}_1 \ldots x_3, {x'}_3) = 4K_3 \frac{\prod_{i=1,3}[\hat{\sigma}(x_i, {x'}_i)G(x_1)G({x'}_1)])}{3\pi^2[S^2 + 3(2+\lambda)R^2]^2}$$

More bound nucleons enter in the game $\rightarrow$ faster the process vanishes at large S.

Two comments

1 Working at fixed fractional momenta a more direct access at the transverse structure is obtained:

Simple example: One-body distribution: $\Gamma(x,b) = G(x)f(b)$

Two-body distribution: $\Gamma(x_1,x_2;b_1,b_2) = G(x_1)G(x_2)K(x_1,x_2)f(b_1,b_2)$

Simple scattering, at fixed x_1, x_2

$$\begin{aligned} \sigma_1^{pp}(x,x') &= G(x)\hat{\sigma}(x,x')G(x') \\ \sigma_2^{pp}(x_1,x_1';x_2,x_2') &= \frac{1}{2}K(x_1,x_2)K(x_1',x_2')\sigma_1^{pp}(x_1,x_1')\sigma_1^{pp}(x_2,x_2')I_1 \end{aligned}$$

Having normalized $f(b)$, the factor I_1 comes from the integration of $f(b_1,b_2)$. We can do the same in proton deuteron:

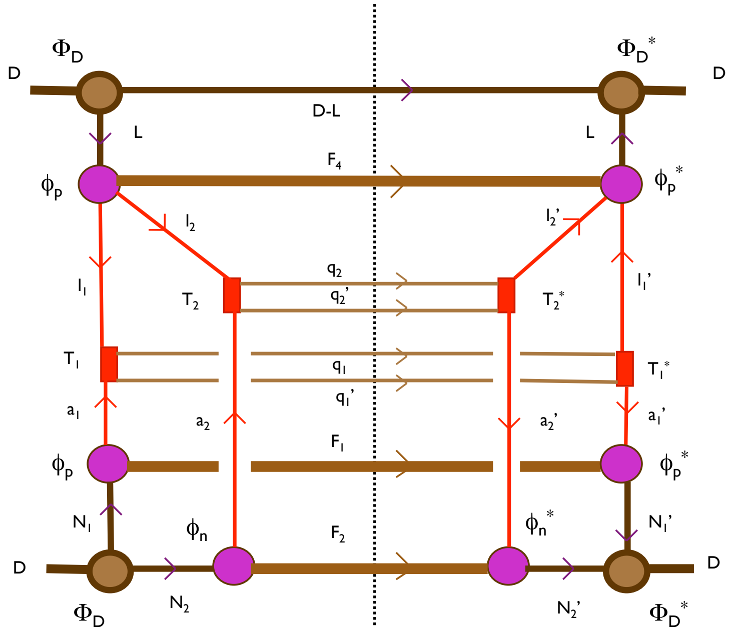
$$\sigma_1^{pD}(x,x') = 2G(x)\hat{\sigma}(x,x')G(x') = 2\sigma_1^{pp}(x,x')$$
$$\sigma_2^{pD}(x_1,x_1';x_2,x_2') = 2\sigma_2^{pp}(x_1,x_1';x_2,x_2') + K(x_1,x_2)\sigma_1^{pp}(x_1,x_1')\sigma_1^{pp}(x_2,x_2')I_2^D$$

I_2^D is the overlap integral between the hadronic and the nuclear distribution.

==================================

2 Other related processes are $D - D$ scattering (double and triple) and also similar processes with triton.

In them there are non new prameters, so they could give consistence tests of the treatment.



One of the three basic graphs for deuteron-deuteron double hard scattering

Here also: More bound nucleons enter $\rightarrow$ faster the process vanishes at large S.

No new parameter enters!

TENTATIVE CONCLUSIONS

Light nuclei provide a tool to analyze the partonic structure of the hadrons.

They allow multiple scatterings in conditions different from hadron-hadron collisions

and so with a different role of the parameters of the distributions.

They are sufficiently simple, they do not blur the fundamental aspects of the interaction:

the main aim is to investigate the many-body aspects of the partonic distributions.

This could be done both in terms of fully integrated quantities

like the effective cross sections and their generalizations to higher multiplicities,

and in term of more differential quantities.

Thank You for Your attention