Final state interaction in kaons decays and $\pi\pi$ scattering lengths measurements.

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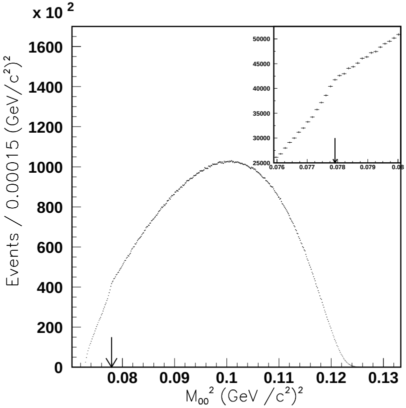
Outline

1. Cusp in $K^+ \to \pi^0\pi^0\pi^+$ decay.
2. Scattering lengths as a test of chiral symmetry breaking.
3. Electromagnetic effects.
4. Scattering lengths extraction from $K^\pm \to \pi^+\pi^- e^\pm \nu$ (K_{e4}) decay.

Cusp in $K^+ \to \pi^0\pi^0\pi^+$ decay.

J.Batley et al.,Phys.Lett. ,2006 Experiment NA48/2, SPS,CERN, 2003-2004, 6×10^7; $K^\pm \to \pi^0\pi^0\pi^\pm$
Anomaly (cusp) in $\pi^0\pi^0$ invariant mass distribution at $M^2 = 4m_c^2$



Two step decay in $K^+ \to \pi^+\pi^0\pi^0$

N.Cabibbo, Phys. Rev. Lett., 2004 Direct $K^+ \to \pi^+\pi^0\pi^0$; Two step contribution as a result of charge exchange in decay $K^+ \to \pi^+\pi^+\pi^-; \pi^+\pi^- \to \pi^0\pi^0$

$$T = T_0 + 2ia_x kT_+; \quad a_x(\pi^+\pi^- \to \pi^0\pi^0) = \frac{a_0 - a_2}{3}$$

$T_0(K^+ \to \pi^+\pi^0\pi^0); \quad T_+(K^+ \to \pi^+\pi^-\pi^+)$ -unperturbed real amplitudes (without final state interaction);
a_0, a_2 are S-wave $\pi\pi$ scattering lengths in the isospin I=0 and I=2 states; $k = \frac{1}{2}\sqrt{M^2 - 4m_c^2}$ - the momenta of charged pions in charge exchange reaction $\pi^+ + \pi^- \to \pi^0 + \pi^0$.
Under charged pions threshold it becomes imaginary $k = i\kappa$

$$
\begin{aligned}
|T|^2 &= T_0^2 + 4\frac{(a_0 - a_2)^2 k^2}{9} T_+^2; M^2 > 4m_c^2 \\
|T|^2 &= T_0^2 + 4\frac{(a_0 - a_2)^2 \kappa^2}{9} T_+^2 - 4\frac{(a_0 - a_2)\kappa}{3} T_0 T_+; M^2 < 4m_c^2
\end{aligned}
$$

Above threshold $M^2 > 4m_c^2$ decay rate $\sim a^2$.
V.N. Gribov Nucl.Phys. (1958) $\tau^+ \to 2\pi^+ + \pi^-$ $\tau' \to 2\pi^0\pi^+$
N.Cabibbo: Under threshold $M^2 < 4m_c^2$ decay rate $\sim a_0 - a_2$
$a_0 = 0.22; a_2 = -0.44$ small quantities.

Scattering lengths as a test of chiral symmetry breaking.

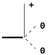
Chiral Perturbation Theory (S.Weinberg,1966) predicts the values of scattering lengths $a_0, a_2 (I = 0, 2)$ with high precision. Weinberg result : $a_0 = \frac{7m^2}{32\pi F_\pi^2} = 0.156$. At present a_0 has been worked out to two loops in the chiral expansion (J.Gasser, H.Leutwyler;1982, J.Gasser et al.1996) and its value is predicted with unusual for strong interaction precision:
$a_0 = 0.22 \pm 0.005$
The values of S-wave scattering lengths a_0, a_2 are of particular interest since they vanish in the chiral limit of zero quark masses. Moreover their values are connected with the size of quark-antiquark vacuum condensate.

Experiment "Dirac", PS, CERN

Determination of pionium ($\pi^+\pi^-$-atom) lifetime. Pionium is unstable through charge exchange reaction $\pi^+\pi^- \to \pi^0\pi^0$.
$\Gamma_{atom} = \frac{1}{\tau} \approx \alpha^3 \mid a_0 - a_2 \mid^2$
The primary goal: Determination of pionium lifetime with $\approx 10\%$ accuracy gives for $a_0 - a_2 \approx 5\%$
At present B.Adeva et al., Phys. Lett., 2005:
$\tau = [2.91 \begin{array}{c} +0.49 \\ -0.62 \end{array}] \times 10^{-15} s; \qquad \mid a_0 - a_2 \mid = 0.264 \begin{array}{c} +0.033 \\ -0.020 \end{array}$
Scattering lengths from NA48/2(J. Batley et al. EPJ, 2009):
$a_0 - a_2 = 0.2571 \pm 0.0087$
(N.Cabbibo, G.Isidori ,JHEP, 2005, J.Gasser et al. Phys.Lett., 2006)

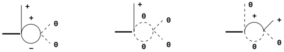
NRQFT: The graphs for $K^+ \longrightarrow \pi^0\pi^0\pi^+$

Tree



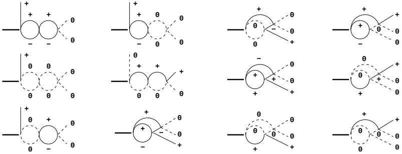
$1, \epsilon^2, \epsilon^4, \ldots$

1-loop



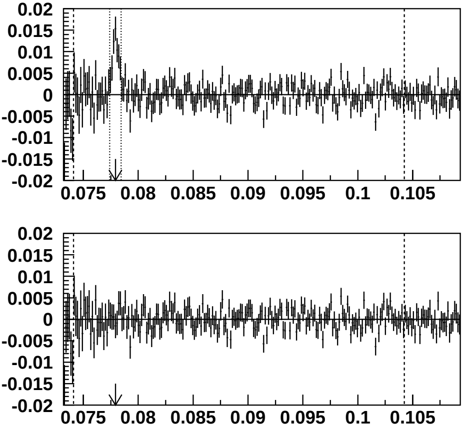
$a\epsilon, a\epsilon^3, a\epsilon^5, \ldots$

2-loops:



$a^2\epsilon^2, a^2\epsilon^4, \cdots$

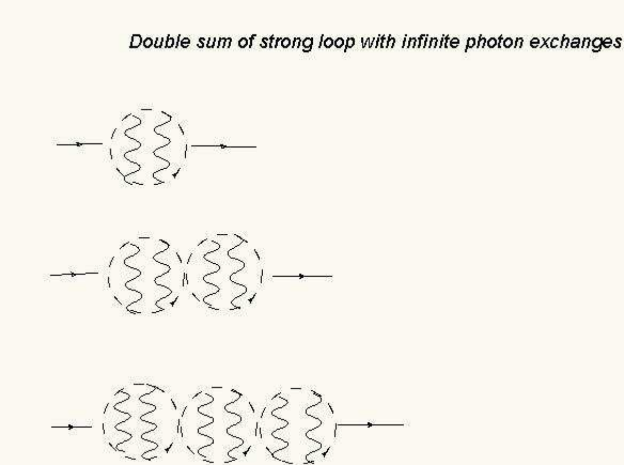
$\Delta = \frac{(data - fit)}{data}$ vs M^2 in Cabibbo -Isidori approach. a)Without pionium; b) with pionium



Higher order terms and Electromagnetic effects in $K^+ \to \pi^0\pi^0\pi^+$

1.Under threshold $M_{\pi\pi} < 2m_c$: Pionium atom just under threshold.
2.Above threshold $M_{\pi\pi} > 2m_c$: Gamov factor.
Two unsolved challenges (2006) :
1.Higher order terms.
2.Electromagnetic effects.
To account for pionium effect NA48/2 multiplied the content of the bin centered at $M^2 = 4m_c^2$ by factor $(1 + f_{atom})$. The best fit value for f_{atom} corresponds to probability of pionium atom production $P = \frac{K^+\to\pi^+ atom}{K^+\to\pi^+\pi^-\pi^+} \approx (1.69 \pm 0.29) \times 10^{-5}$, whereas the theory predict (Silagadze, JETP Letters, 1994) $P = 0.8 \times 10^{-5}$

Unstable atoms and electromagnetic effects



Strong interactions to all orders in $K \longrightarrow 2\pi$

S.Gevorkyan,A.Tarasov,O.Voskresenskaya, Eur. Phys. J. 2010

$$M_c(K \to \pi^+\pi^-) = \int \Psi_c^+(r) M_0(r) d^3r; k_c = \frac{\sqrt{M^2 - 4m_c^2}}{2}$$
$$M_n(K \to \pi^0\pi^0) = \int \Psi_n^+(r) M_0(r) d^3r; k_n = \frac{\sqrt{M^2 - 4m_n^2}}{2}$$

$$-\Delta\Psi_c(r) + U_{cc}\Psi_c(r) + U_{cn}\Psi_n(r) = k_c^2\Psi_c(r);$$
$$-\Delta\Psi_n(r) + U_{nn}\Psi_n(r) + U_{nc}\Psi_c(r) = k_n^2\Psi_n(r)$$

S-wave scattering, strong potential with sharp boundary $U_{ik} \gg k_{c(n)}$, asymptotic behavior of wave functions, unitarity constrains.

Final result. Strong $\pi\pi$ interaction to all orders.

The decay amplitudes $K \to 2\pi$ are expressed through the "unperturbed" decay amplitudes M_{0c}, M_{0n} and $\pi\pi$ scattering amplitudes

$f_x(\pi^+\pi^- \to \pi^0\pi^0), \quad f_c(\pi^+\pi^- \to \pi^+\pi^-), \quad f_n(\pi^0\pi^0 \to \pi^0\pi^0)$

$$M_c = M_{0c}(1 + ik_c f_{cc}) + ik_n M_{0n} f_x; f_{cc} = \frac{a_{cc}(1 - ik_n a_{nn}) + ik_n a_x^2}{D}$$

$$M_n = M_{0n}(1 + ik_n f_{nn}) + ik_c M_{0c} f_x; f_{nn} = \frac{a_{nn}(1 - ik_c a_{cc}) + ik_c a_x^2}{D}$$

$$f_x = \frac{a_x}{D}; D = (1 - ik_c a_{cc})(1 - ik_n a_{nn}) + k_n k_c a_x^2$$

In the case of exact isospin symmetry in the vicinity of threshold:

$$a_x = \frac{a_0 - a_2}{3}; \quad a_{nn} = \frac{a_0 + 2a_2}{3}; \quad a_{cc} = \frac{2a_0 + a_2}{6}$$

Electromagnetic interaction in $\pi^+\pi^-$ system

S.Gevorkyan,A.Tarasov,O.Voskresenskaya, Phys. Lett. 2007
Receipt is known for many years (Wigner, 1948, Breit, 1957, Baz , 1957) $ik_c \to \tau = \frac{d\log[G_0(kr)+iF_0(kr)]}{dr}\Big|_{r=r_0}$ F_0, G_0 are the regular and irregular solutions of the Coulomb problem (Hypergeometric functions). In the region $kr_0 \ll 1$:

$$\tau = ik - \alpha m\left[\log(-2ikr_0) + 2\gamma + \psi(1-i\xi)\right]$$

$$Re\ \tau = -\alpha m\left[\log(2kr_0) + 2\gamma + Re\,\psi(1-i\xi)\right], \xi = \frac{\alpha m}{2k}$$

$$Im\ \tau = kA^2, \quad A = \exp\left(\frac{\pi\xi}{2}\right)|\Gamma(1+i\xi)|$$

$\gamma = 0.5772$; digamma function $\psi(\xi) = \frac{d\log\Gamma(\xi)}{d\xi}$

Final results. Electromagnetic interaction between pions to all orders.

1.Under threshold $M_{\pi\pi} < 2m_c$: Pionium atom just under charged pions threshold.

S.Gevorkyan,D.Madigozhin,A.Tarasov,O.Voskresenskaya, Phys.Part.Nucl. Lett.,2008

The contribution of pionium belong to the central bin and coincides with Silagadze prediction 0.8×10^{-5}. The excess in NA48/2 is connected with $\pi\pi$ proper electromagnetic interaction.

2.Above threshold $M_{\pi\pi} > 2m_c$: Gamov-Zommerfield factor

$G = \frac{2\pi\omega}{1-e^{-2\pi\omega}}; \quad \omega = \frac{\alpha}{\upsilon}$

Decay with two charged pions in final state: $K^{\pm} \to \pi^{+}\pi^{-}e^{\pm}\nu$ ($K_{e4} decay$)

(Rosselet et al.,CERN ,1977 ($a_0 = 0.26 \pm 0.05$); Pislak et al. BNL ,2003 ($a_0 = 0.216 \pm 0.017$))
NA48/2,CERN Bloch-Devaux, Kaon workshops: December 2006; May,September 2007 $\sim 10^6$ events
$a_0^0 = 0.256 \pm 0.006 \pm 0.005$
ChPT $a_0^0 = 0.22 \pm 0.005$
1.The isospin symmetry breaking effects in K_{e4} decays.
2.The electromagnetic effects in K_{e4} decay.

Short introduction to $K^{\pm} \to \pi^+\pi^- e^{\pm}\nu$ decay

$$M = \frac{G_F \sin\theta_c}{\sqrt{2}} \langle \pi^+\pi^- | J^{\mu}_{had} | K^+ \rangle \langle e^+\nu_e | J^{lep}_{\mu} | 0 \rangle .$$

$$F = f_s e^{i\delta_0^0(s)} + f_p e^{i\delta_1^1(s)} \cos\theta_\pi ; G = g_p e^{i\delta_1^1(s)} ; H = h_p e^{i\delta_1^1(s)}$$

$\delta_0^0(s), \delta_1^1(s)$-phases in $\pi\pi$ scattering (I=l=0 and I=l=1)
Fermi-Watson theorem: $\delta_0^0(s); \delta_1^1(s) \to \delta(\pi^+\pi^- \to \pi^+\pi^-)$
$\tan\delta_l^I = \sqrt{1 - \frac{4m_c^2}{s}} q^{2l} [A_l^I + B_l^I q^2 + C_l^I q^4 + D_l^I q^6]$
$q^2 = \frac{s-4m^2}{4}$
Roy equations connect these coefficients with scattering lengths:
$A_l^I = A_l^I(a_0^0, a_0^2)....$
From experiment the difference $\delta = \delta_0^0 - \delta_1^1$ is extracted.

Charge exchange in K_{e4}

S.Gevorkyan, A.Sissakian, A.Tarasov, H.Torosyan, O.Voskresenskaya Phys.Atom.Nucl.,2010; hep-ph/0704.2675

We take into account the charge exchange (a la N.Cabibbo) $K \to \pi^0\pi^0 e\nu \to \pi^+\pi^- e\nu$ For dipion state I=I=0

$$
\begin{aligned}
M &= M_c(1 + ik_c a_c) + ik_n a_x M_n; \, a_c = \frac{2a_0 + a_2}{3}(1+\epsilon); \\
a_x &= \frac{1}{3}(a_0 - a_2)(1 + \epsilon/3); \, \epsilon = \frac{m_c^2 - m_n^2}{m_c^2}
\end{aligned}
$$

$$
\delta_0^0 = \arctan \frac{k_n a_n + k_c a_c}{1 + k_n k_c (a_x^2 - a_n a_c)} - \arctan k_n \left(a_n - \frac{1}{\sqrt{2}} a_x\right)
$$

Modified phases by electromagnetic effects

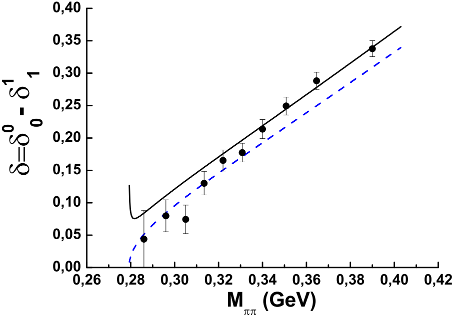
S.Gevorkyan, A.Sissakian, A.Tarasov, H.Torosyan, O.Voskresenskaya Phys.Atom.Nucl.,2010; hep-ph/0711.4618

Strong δ_{str} and electromagnetic δ_{em} :

$$\begin{aligned}
\tilde{\delta}_0^0 &= \delta_{str} + \delta_{em} \quad \delta_{str} = \arctan(A \tan \delta_0^0 + B \tan \delta_0^2); \\
\delta_{em} &= \arctan(\frac{\alpha}{\beta}); \quad \beta = \frac{\sqrt{1-4\upsilon}}{1-2\upsilon} \\
A &= \frac{2G(1+\epsilon) + \lambda(1+\frac{\epsilon}{3})}{3}; \quad B = \frac{G(1+\epsilon) - \lambda(1+\frac{\epsilon}{3})}{3} \\
G &= \frac{2\pi\omega}{1-e^{-2\pi\omega}}; \quad \omega = \frac{\alpha}{\upsilon} \\
\lambda &= \sqrt{\frac{1-4u_0}{1-4u_c}} \quad u_c = \frac{m_c^2}{s}; \quad u_0 = \frac{m_n^2}{s}
\end{aligned}$$

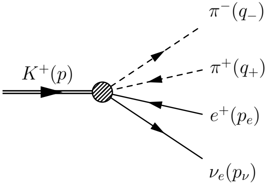
$$\tilde{\delta}_1^1 = \arctan\left(G(1+\frac{\alpha^2}{\beta^2}) \tan \delta_1^1\right). \qquad (1)$$

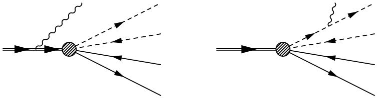
Modified and usual phases dependence on invariant mass



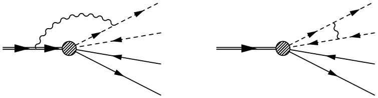
Radiative corrections in K_{e4} decay

Yu.Bystritskiy, S. Gevorkyan, E. Kuraev Eur. Phys. J. 2009





a.) *b.)*



a.) *b.)*

K_{e4} decay rate with radiative corrections

$$\frac{d\Gamma}{d\Gamma_B} = G(\omega)(1+\frac{\alpha}{\pi}K)F(\frac{s_l}{s_{max}},\sigma);\sigma = \frac{\alpha}{2\pi}(2\ln(\frac{2E_e}{m_e})-1)$$

$$F(z,\sigma) = 2\sigma(1+\frac{3}{2}\sigma)\int_z^1 \frac{dx}{x}(1-x)^{2\sigma-1} - \sigma\ln\frac{1}{z} + 1 - z$$