

# Weak neutrinos–matter interactions due to contraction of Lepton Sector of the Electroweak Model

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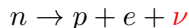
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# Outline

- ▶ Neutrino: main experimental properties.
- ▶ The Standard Electroweak Model.
- ▶ The Modified Model.
- ▶ Conclusion

## Neutrino: main experimental properties

- ▶ During the last century many elementary particles were discovered and studied. One of these the **neutrino** was theoretically introduced by W. Pauli in 1930 under analysis of neutron beta decay



- ▶ The name **neutrino** was given by E. Fermi a little later.
- ▶ The **electron neutrino** was experimentally verified in 1956.
- ▶ The **muon neutrino** was discovered in 1962.
- ▶ The last one **tau neutrino** was detected in 1975.
- ▶ According to the Standard Electroweak Model there are only three sorts of neutrinos (1989).

- ▶ All neutrinos are stable. They are **weak** and **gravity** interacted particles.
- ▶ For a long time neutrino was considered as massless particle. But recently experimental indications appeared that neutrino has very small mass

$$m_\nu < 1 eV.$$

For comparison: electron mass

$$m_e = 0,5 MeV = 500\,000 eV.$$

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- ▶ The **vanishingly small interaction with matter** especially for low energies distinguishes neutrinos from other elementary particles. The attenuation length of neutrino in water is about 100 light years.
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- ▶ In this talk we suggest the modification of the Standard Electroweak Model which explain this fact already **at classical (non quantum) level**.

## The Standard Electroweak Model

- ▶ **First phenomenological theory of the weak interactions (the theory of beta decay) was developed by E. Fermi in 1934.**
- ▶ Weak interactions for low energies are characterized by the Fermi constant  $G_F$ . This constant is determined by experimental measurements and turn out to be very small  $G_F = 10^{-5} \frac{1}{m_p^2} = 1,17 \cdot 10^{-5} GeV^{-2}$ .
- ▶ Modern theory, which unified weak and electromagnetic interactions, was suggested in 1968 by S. Weinberg, S.L. Glashow, A. Salam and is known as the Standard Electroweak Model.
- ▶ This model is a gauge theory with the gauge group  $SU(2) \times U(1)$ , which act in the boson, lepton and quark sectors.



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- ▶ The Standard Model is important to theoretical and experimental particle physics because of its success in explaining a wide variety of experimental results.
- ▶ Due to this model the  $W$ - and  $Z$ -bosons was predicted and experimentally observed at the end of the last century. Higgs boson is now searched at the LHC.
- ▶ At the same time the grave disadvantage of the Standard Model is the presence more then fifteen free parameters.
- ▶ But among these there is not such parameter, which **a priori** can be regarded as **a small one** and can be connected with the very rare interaction neutrinos with matter.

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► Elementary particles content of the Standard Model.

Gauge bosons:

$\gamma$  (photon),  $W^\pm$  (charged weak bosons),  
 $Z^0$  (neutral weak boson).

Quarks:

$$\begin{pmatrix} u \\ d \end{pmatrix}, \quad \begin{pmatrix} c \\ s \end{pmatrix}, \quad \begin{pmatrix} t \\ b \end{pmatrix} \in C_2.$$

Leptons:

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}, \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}, \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \in C_2.$$



- ▶ The Lagrangian of the model is given by the sum:

$$L = L_B + L_Q + L_L$$

of the boson  $L_B$ , of the quark  $L_Q$  and of the lepton  $L_L$  Lagrangians

- ▶ and is invariant under the action of the gauge group  $SU(2) \times U(1)$  in  $C_2$

$$\begin{pmatrix} z'_1 \\ z'_2 \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ -\bar{\beta} & \bar{\alpha} \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}, \quad |\alpha|^2 + |\beta|^2 = 1.$$

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- ▶ The boson  $L_B$  and the quark  $L_Q$  Lagrangians are not changed in our model, so we concentrate our attention on **the lepton Lagrangian**, which for the first lepton generation is written in the form:

$$L_{L,e} = L_l^\dagger i \tilde{\tau}_\mu D_\mu L_l + e_r^\dagger i \tau_\mu D_\mu e_r - h_e [e_r^\dagger (\phi^\dagger L_l) + (L_l^\dagger \phi) e_r],$$

- ▶ where  $L_l = \begin{pmatrix} \nu_l \\ e_l \end{pmatrix} \in C_2$  is the  $SU(2)$ -doublet,  $e_r$  is the  $SU(2)$ -singlet,  $h_e$  is constant,  $\tau_0 = \tilde{\tau}_0 = \mathbf{1}$ ,  $\tilde{\tau}_k = -\tau_k$ ,  $\tau_\mu$  are the Pauli matrices,  $\phi \in C_2$  are the matter fields and  $e_r, e_l, \nu_l$  are the two component Lorentzian spinors.
- ▶ First and second terms in  $L_{L,e}$  describe free movement of left and right fermions and their interactions with gauge fields.
- ▶ Last term corresponds to the electron mass.

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- ▶ The covariant derivatives of the lepton fields are given by the formulas:

$$D_\mu e_r = \partial_\mu e_r + ig' A_\mu e_r \cos \theta_w - ig' Z_\mu e_r \sin \theta_w,$$

$$D_\mu L_l = \partial_\mu L_l - i \frac{g}{\sqrt{2}} (W_\mu^+ T_+ + W_\mu^- T_-) L_l -$$

$$-i \frac{g}{\cos \theta_w} Z_\mu (T_3 - Q \sin^2 \theta_w) L_l - ie A_\mu Q L_l,$$

- ▶ where  $T_k = \frac{1}{2} \tau_k$ ,  $k = 1, 2, 3$  are the generators of  $SU(2)$ ,  
 $T_\pm = T_1 \pm iT_2$ ,  
 $Q = Y + T_3$  is the generator of electromagnetic subgroup  $U(1)_{em}$ ,  
 $Y = -\frac{1}{2} \mathbf{1}$  is the hypercharge of the left leptons,  
 $e = \frac{gg'}{\sqrt{g^2 + g'^2}}$  is the electron charge and  $\sin \theta_w = \frac{e}{g}$ .

► The new gauge fields

$$W_{\mu}^{\pm} = \frac{1}{\sqrt{2}} (A_{\mu}^1 \mp iA_{\mu}^2), \quad Z_{\mu} = \frac{1}{\sqrt{g^2 + g'^2}} (gA_{\mu}^3 - g'B_{\mu}),$$

$$A_{\mu} = \frac{1}{\sqrt{g^2 + g'^2}} (g'A_{\mu}^3 + gB_{\mu})$$

► are expressed through the fields

$$A_{\mu}(x) = -ig \sum_{k=1}^3 T_k A_{\mu}^k(x), \quad B_{\mu}(x) = -ig' B_{\mu}(x),$$

which take their values in the Lie algebras  $su(2)$  and  $u(1)$ , respectively.



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- ▶ The next two lepton generations (muon and muon neutrino,  $\tau$ -lepton and  $\tau$ -neutrino) are introduced in a similar way.
- ▶ Full lepton Lagrangian is the sum:

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## Modified Model

- ▶ In the **Standard Electroweak Model** the gauge group  $SU(2) \times U(1)$  acts in the boson, lepton and quark sectors, i.e. for the lepton fields

$$\begin{pmatrix} \nu'_e \\ e' \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ -\bar{\beta} & \bar{\alpha} \end{pmatrix} \begin{pmatrix} \nu_e \\ e \end{pmatrix}, \quad |\alpha|^2 + |\beta|^2 = 1.$$

- ▶ Let us **consistently rescale the group  $SU(2)$  and field space  $C_2$**  in the lepton sector by the parameter  $j \in \mathbb{R}$

$$\begin{pmatrix} j\nu'_e \\ e' \end{pmatrix} = \begin{pmatrix} \alpha & j\beta \\ -j\bar{\beta} & \bar{\alpha} \end{pmatrix} \begin{pmatrix} j\nu_e \\ e \end{pmatrix}, \quad |\alpha|^2 + j^2|\beta|^2 = 1.$$

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- ▶ **The left lepton fields** are now as follows

$$\begin{pmatrix} j\nu_e \\ e \end{pmatrix}, \begin{pmatrix} j\nu_\mu \\ \mu \end{pmatrix}, \begin{pmatrix} j\nu_\tau \\ \tau \end{pmatrix} \in C_2(j).$$

- ▶ They are obtained from those of the Standard Model by multiplication of the neutrino fields on parameter  $j$

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- ▶ Substitution  $\beta \rightarrow j\beta$ , induces another ones for  $su(2)$  algebra generators

$$T_1 \rightarrow jT_1, \quad T_2 \rightarrow jT_2, \quad T_3 \rightarrow T_3.$$

- ▶ As far as the gauge fields take their values in Lie algebra, we can **rescale the gauge fields** instead of transforming the generators, namely:

$$A_\mu^1 \rightarrow jA_\mu^1, \quad A_\mu^2 \rightarrow jA_\mu^2, \quad A_\mu^3 \rightarrow A_\mu^3, \quad B_\mu \rightarrow B_\mu.$$

- ▶ Indeed, due to commutativity and associativity of multiplication by  $j$

$$\begin{aligned} g(j) &= \exp \{ A_\mu^1(jT_1) + A_\mu^2(jT_2) + A_\mu^3 T_3 \} = \\ &= \exp \{ (jA_\mu^1)T_1 + (jA_\mu^2)T_2 + A_\mu^3 T_3 \}. \end{aligned}$$

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- ▶ Above transformations of the neutrinos and gauge fields give rise to the lepton Lagrangian

$$\begin{aligned}
 L_{L,e}(j) = & e_l^\dagger i \tilde{\tau}_\mu \partial_\mu e_l + e_r^\dagger i \tau_\mu \partial_\mu e_r - e e_l^\dagger \tilde{\tau}_\mu A_\mu e_l + \frac{g \cos 2\theta_w}{2 \cos \theta_w} e_l^\dagger \tilde{\tau}_\mu Z_\mu e_l - \\
 & - g' \cos \theta_w e_r^\dagger \tau_\mu A_\mu e_r + g' \sin \theta_w e_r^\dagger \tau_\mu Z_\mu e_r - m_e [e_r^\dagger e_l + e_l^\dagger e_r] + \\
 & + j^2 \left\{ \nu_l^\dagger i \tilde{\tau}_\mu \partial_\mu \nu_l + \frac{g}{2 \cos \theta_w} \nu_l^\dagger \tilde{\tau}_\mu Z_\mu \nu_l + \right. \\
 & \left. + \frac{g}{\sqrt{2}} [\nu_l^\dagger \tilde{\tau}_\mu W_\mu^+ e_l + e_l^\dagger \tilde{\tau}_\mu W_\mu^- \nu_l] \right\} = L_{e,b} + j^2 L_{\nu,f}.
 \end{aligned}$$

- ▶ Lagrangian  $j^2 L_{\nu,f}$  describe free neutrino and its interactions with electron and gauge fields.
- ▶ Lagrangian  $L_{e,b}$  describe free movement of left and right electrons as well as their interactions with gauge fields and electron mass.

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- Lagrangian  $j^2 L_{\nu,f}$  describe free neutrino and its interactions with electron and gauge fields.
- Lagrangian  $L_{e,b}$  describe free movement of left and right electrons as well as their interactions with gauge fields and electron mass.



- ▶ Above transformations of the neutrinos and gauge fields give rise to the lepton Lagrangian

$$\begin{aligned}
 L_{L,e}(j) = & e_l^\dagger i \tilde{\tau}_\mu \partial_\mu e_l + e_r^\dagger i \tau_\mu \partial_\mu e_r - e e_l^\dagger \tilde{\tau}_\mu A_\mu e_l + \frac{g \cos 2\theta_w}{2 \cos \theta_w} e_l^\dagger \tilde{\tau}_\mu Z_\mu e_l - \\
 & - g' \cos \theta_w e_r^\dagger \tau_\mu A_\mu e_r + g' \sin \theta_w e_r^\dagger \tau_\mu Z_\mu e_r - m_e [e_r^\dagger e_l + e_l^\dagger e_r] + \\
 & + j^2 \left\{ \nu_l^\dagger i \tilde{\tau}_\mu \partial_\mu \nu_l + \frac{g}{2 \cos \theta_w} \nu_l^\dagger \tilde{\tau}_\mu Z_\mu \nu_l + \right. \\
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- ▶ We put  $\phi = \phi^{vac} = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$  in  $L_{L,e}$  and denote the electron mass as  $m_e = \frac{h_e v}{\sqrt{2}}$ .
- ▶ Next lepton generation fields are transformed in a similar way:

$$\nu_{\mu,l} \rightarrow j\nu_{\mu,l}, \quad \nu_{\tau,l} \rightarrow j\nu_{\tau,l}, \quad \mu_l \rightarrow \mu_l, \quad \tau_l \rightarrow \tau_l,$$

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- ▶ The lepton Lagrangian is the sum:

$$L_L(j) = L_{L,e}(j) + L_{L,\mu}(j) + L_{L,\tau}(j) = L_{L,b} + j^2 L_{L,f},$$

where each term has the structure of  $L_{L,e}(j)$  with the mass  $m_q = \frac{h_q v}{\sqrt{2}}$ ,  $q = e, \mu, \tau$ .

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► **The full Lagrangian of the model**

$$L(j) = L_B + L_Q + L_L(j) = L_r + j^2 L_\nu$$

**is split in two parts:**

- the Lagrangian  $j^2 L_\nu$ , which includes neutrino fields along with their interactions with gauge and lepton fields and
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## The mathematical interpretation of the parameter $j$

- ▶ To understand the meaning of the parameter  $j$  let us consider the limit  $j \rightarrow 0$  in the action of  $SU(2; j)$  on the space  $C_2(j)$

$$\begin{pmatrix} j\nu'_e \\ e' \end{pmatrix} = \begin{pmatrix} \alpha & j\beta \\ -j\bar{\beta} & \bar{\alpha} \end{pmatrix} \begin{pmatrix} j\nu_e \\ e \end{pmatrix}, \quad |\alpha|^2 + j^2|\beta|^2 = 1.$$

- ▶ In this limit (or for  $j = \iota$ ,  $\iota^2 = 0$ ) the unitary group  $SU(2; j)$  is contracted to the motion group  $E(2)$  of the Euclid plane.
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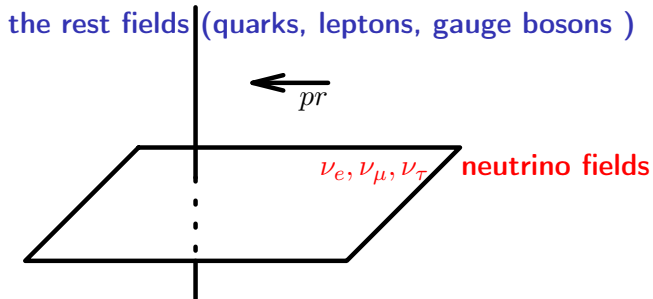
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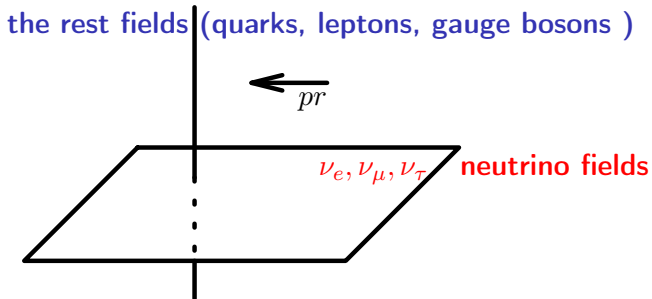
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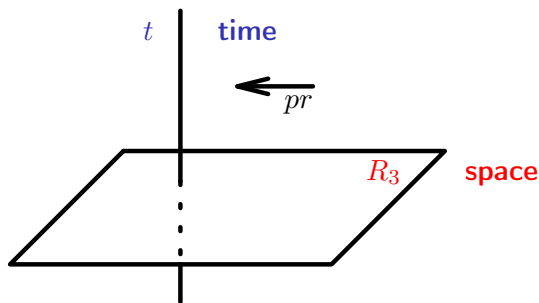
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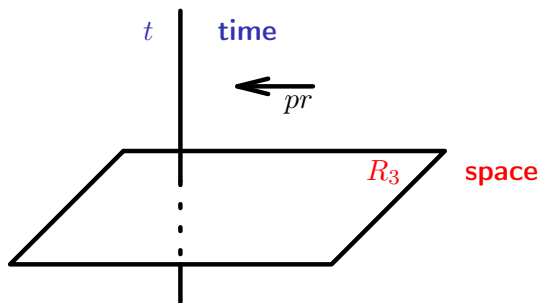
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## The physical interpretation of the parameter $j$

- ▶ The space-time of the special relativity is transformed to the nonrelativistic space-time when **dimensionfull** contraction parameter — velocity of light  $c$  — tends to infinity and **dimensionless** parameter  $\frac{v}{c} \rightarrow 0$ .
- ▶ From the neutrino part Lagrangian

$$j^2 L_{\nu,f} = j^2 \left\{ \nu_l^\dagger i \tilde{\tau}_\mu \partial_\mu \nu_l + \frac{g}{2 \cos \theta_w} \nu_l^\dagger \tilde{\tau}_\mu Z_\mu \nu_l + \right. \\ \left. + \frac{g}{\sqrt{2}} \left[ \nu_l^\dagger \tilde{\tau}_\mu W_\mu^+ e_l + e_l^\dagger \tilde{\tau}_\mu W_\mu^- \nu_l \right] \right\}$$

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- ▶ Therefore probability amplitudes for weak current interactions, which include such vertexes, for example

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are multiplied by the same factor  $j^2$ .

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- ▶ If we suppose that parameter  $j$  depend on neutrino energy in an appropriate manner, then contraction procedure can be connected with the dependence of neutrino-matter cross-section on neutrino energy, which is decreased when neutrino energy go down.

## Conclusion

The suggested model explain the very weak neutrinos-matter interactions on the level of classical fields with the help of contraction of the gauge group in the lepton sector of the Standard Electroweak Model, leaving the invariance group in the boson and quark sectors untouched.

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The very small Fermi constant is represented

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by the contraction parameter, which is small by implication.

Thank you  
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