NEW FORMULATION OF ELECTROWEAK MODELS

APPLICABLE BEYOND PERTURBATION THEORY.

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A new symmetry usually allows to construct more transparent formulation of the theory.

Recent examples are given by gauge theories. QED may be formulated in the Coulomb gauge, however much more transparent formulation is presented by the quantization in a manifestly covariant gauge, which is possible due to the gauge invariance of the theory. Yang-Mills theory became really popular only after its formulation in the Lorentz covariant terms and explicit proof of its renormalizability which was possible because of the gauge invariance. The gauge invariance of the Higgs model allows to give a manifestly renormalizable theory describing a massive gauge theory.

In this talk I wish to make a propaganda for a new class of symmetries, which were introduced in my paper rather long ago (A.A.S., 1991), but recently were applied successfully to the nonperturbative quantization of non-Abelian gauge theories.

Equivalence theorems: canonical transformations,
point transformations $\varphi = \varphi' + f(\varphi')$

More general transformations:

$$\varphi = \frac{\partial^n \varphi'}{\partial t^n} + f(\frac{\partial^{n-1}\varphi'}{\partial t^{n-1}}, \ldots \frac{\partial \varphi'}{\partial t}) = \tilde{f}(\varphi') \qquad (1)$$

The spectrum is changed. What about the unitarity?

Path integral representation for the scattering matrix

$$S = \int \exp\{i \int L(\varphi)dx\} d\mu(\varphi); \quad lim_{t \to \pm\infty} \varphi(x) = \varphi_{out,in}(x) \tag{2}$$

If the change (1) does not change the asymptotic conditions, then the only effect of such transformation is the appearance of a nontrivial jacobian

$$L(\varphi) \to \tilde{L}(\varphi') = L[\varphi(\varphi')] + \bar{c}^a \frac{\delta \varphi^a}{\delta \varphi'^b} c^b \tag{3}$$

For all new excitations one should take the vacuum boundary conditions.

Unitarity?

The new Lagrangian is invariant with respect to the supertransformations

$$\delta\varphi'_a = c_a\varepsilon$$
$$\delta c_a = 0; \quad \delta\bar{c}_a = \frac{\delta L}{\delta\varphi_a}(\varphi')\varepsilon \tag{4}$$

On mass shell these transformations are nilpotent and generate a conserved charge Q. In this case there exists an invariant subspace of states annihilated by Q, which has a semidefinite norm. (A.A.S.,1991). For asymptotic space this condition reduces to

$$Q_0|\phi >_{as}= 0 \tag{5}$$

The scattering matrix is unitary in the subspace which contains only excitations of the original theory. However the theories described by the L and the $\tilde{L}$ are different, and only expectation values of the gauge invariant operators coincide. In gauge theories the transition from one gauge to another may be considered as such a change.

A very nontrivial generalization is obtained if one transforms the $\tilde{L}$ further shifting the fields φ' by constants. It is not an allowed change of variables in the path integral as it changes the asymptotic of the fields. The unitarity of the "shifted"theory is not guaranteed and a special proof (if possible) is needed.

Using this method one can construct a renormalizable formulation of nonabelian gauge theories free of the Gribov ambiguity.

A.A.Slavnov,JHEP,0808(2008)047; Theor.Math.Phys,161(2009)204; A.Quadri, A.A.Slavnov, JHEP, 1007 (2010); A.Quadri, A.A.Slavnov, Theor.Math.Phys, 166(2011)201

A problem of unambiguos quantization of nonabelian gauge theories beyond perturbation theory remains unsolved. Even in classical theory the equation

$$D_\mu F_{\mu\nu} = 0 \qquad (6)$$

does not determine the Cauchi problem. Gauge invariance results in existence of many solutions of this equation. To define the classical Cauchi problem and subsequently to quantize the model one imposes a gauge condition, e.g. Coulomb gauge $\partial_i A_i = 0$.

Differential gauge conditions: $L(A_\mu, \varphi) = 0 \to$ Gribov ambiguity.

Algebraic gauge conditions: $\tilde{L}(A_\mu, \varphi) = 0 \to$ absence of the manifest Lorentz invariance and other problems.

Coulomb gauge

$$\begin{aligned} \partial_i A_i &= 0 \\ A'_i &= (A^{\Omega})_i \\ \triangle\alpha^a + ig\varepsilon^{abc}\partial_i(A^b_i\alpha^c) &= 0 \end{aligned} \tag{7}$$

This equation has nontrivial solutions decreasing at spatial infinity$\rightarrow$Gribov ambiguity.

In perturbation theory the only solution is $\alpha = 0$.

Weinberg-Salam model

$$L = -1/4 F^a_{\mu\nu} F^a_{\mu\nu} - 1/4 G^a_{\mu\nu} G^a_{\mu\nu} + i\bar{L}\gamma^\mu (\partial_\mu + \frac{ig}{2}\tau^a A^a_\mu + \frac{ig_1}{2} B_\mu) L$$
$$+ i\bar{R}\gamma_\mu (\partial_\mu + ig_1 B_\mu) R + |\partial_\mu \varphi + \frac{ig}{2}\tau^a A^a_\mu \varphi + \frac{ig_1}{2} B_\mu \varphi|^2 -$$
$$-G\{(\bar{L}\varphi)R + \bar{R}(\varphi^* L)\} + \frac{m^2}{2}(\varphi^*\varphi) - \lambda^2(\varphi^*\varphi)^2 \quad (8)$$

where

$$\varphi(x) = (\varphi_1(x), \varphi_2(x)) = \sqrt{2}^{-1}(iB_1 + B_2, \sigma - iB_3 + \sqrt{2}\mu) \quad (9)$$

In perturbation theory all predictions fit the experiment very well.

However there are certain questions to be answered

1. Where is the Higgs meson? (LHC).

2. Is the model valid beyond perturbation theory?

3. Is it possible to derive the Weinberg-Salam model from some grand-unified model?

4. Quantization of the Weinberg-Salam model beyond the perturbation theory?

$SU(2)$ **Higgs-Kibble model**

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$$L = -1/4 F^a_{\mu\nu} F^a_{\mu\nu} + (D_\mu \varphi)^* (D_\mu \varphi) - \lambda^2 (\varphi^* \varphi - \mu^2)^2 \tag{10}$$

Gauge transformations:

$$\begin{aligned} \delta A^a_\mu &= \partial_\mu \eta^a + g \varepsilon^{abc} A^b_\mu \eta^c, \\ \delta B^a &= \mu \sqrt{2} \eta^a + \frac{g}{2} \varepsilon^{abc} B^b \eta^c + \frac{g}{2} \sigma \eta^a. \end{aligned} \tag{11}$$

Unitary gauge $B^a = 0$ The spectrum: Three components of the massive vector field A^a_i.

One scalar field (Higgs meson) σ.

Unitarity is obvious, but there is no renormalizability.

Renormalizable gauges: $\partial_\mu A^a_\mu = 0$.

The spectrum:

A^a_i, σ, unphysical components of A^a_μ, Faddeev-Popov ghosts $\bar{c}^a, c^a$, Goldstone bosons B^a.

The unitarity in the physical subspace should be proven!

Nonuniqueness of the gauge fixing does not allow to do that beyond perturbation theory.

An alternative formulation of the Higgs-Kibble model.

$$
\begin{aligned}
L = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + (D_\mu\varphi^+)^*(D_\mu\varphi^-) + (D_\mu\varphi^-)^*(D_\mu\varphi^+) \\
+(D_\mu\varphi)^*(D_\mu\varphi) - \lambda^2(\varphi^*\varphi - \mu^2)^2 \\
-[(D_\mu b)^*(D_\mu e) + (D_\mu e)^*(D_\mu b)]
\end{aligned}
\qquad (12)
$$

Here the field φ is the complex doublet describing the Higgs meson, and the fields $\varphi^\pm$ are new auxiliary fields. The fields b, e have a similar structure, but correspond to the anticommuting fields. The shift

$$
\varphi^-(x) \to \varphi^-(x) - \widehat{m}; \quad \varphi(x) \to \varphi(x) - \widehat{\mu} \qquad (13)
$$

where $\widehat{m}$ and $\widehat{\mu}$ are the coordinate-independent condensates

$$
\widehat{m} = (0, m/g); \quad \widehat{\mu} = (0, \mu/g) \qquad (14)
$$

generates the mass term for the vector field.

The new Lagrangian describing the massive vector field is

$$
\begin{aligned}
L = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + (D_\mu\varphi^+)^*(D_\mu\varphi^-) + (D_\mu\varphi^-)^*(D_\mu\varphi^+) \\
-[(D_\mu\varphi^+)^*(D_\mu\widehat{m}) + (D_\mu\widehat{m})^* D_\mu\varphi^+] \\
-[(D_\mu b)^*(D_\mu e) + (D_\mu e)^*(D_\mu b)] + (D_\mu\varphi)^*(D_\mu\varphi) \\
-[(D_\mu\varphi)^*(D_\mu\widehat{\mu}) + (D_\mu\widehat{\mu})^*(D_\mu\varphi)] \\
+(D_\mu\widehat{\mu})^*(D_\mu\widehat{\mu}) - \lambda^2[(\varphi-\widehat{\mu})^*(\varphi-\widehat{\mu}) - \mu^2]^2 .
\end{aligned}
\tag{15}
$$

After the shift both the fields φ and φ_- become the gauge fields:

$$
\begin{aligned}
\delta\varphi^a_- &= m\eta^a + \frac{g}{2}\epsilon^{abc}\varphi^b_-\eta^c + \frac{g}{2}\varphi^0_-\eta^a \\
\delta\varphi^a &= \mu\eta^a + \frac{g}{2}\epsilon^{abc}\varphi^b\eta^c + \frac{g}{2}\varphi^0\eta^a
\end{aligned}
\tag{16}
$$

A gauge condition may be imposed on the fields $A^a_\mu, \varphi^a, \varphi^a_-$.

We choose the gauge $\varphi_{-}^{a} = 0$

This is an algebraic gauge, which is manifestly Lorentz invariant and, as we shall see, renormalizable.

An ambiguity?

$$\varphi_{-}^{a} = 0; \quad (\varphi_{-}^{\Omega})^{a} = 0; \quad (\varphi_{-}^{\Omega})^{a} = \varphi_{-}^{a} + (m + \frac{g}{2}\varphi_{-}^{0})\eta^{a} \tag{17}$$

For large φ_{-}^{0} there is an ambiguity

This ambiguity may be eliminated by a simple change of variables in the classical Lagrangian

$$\varphi^0_- = \frac{2m}{g}(\exp\{\frac{gh}{2m}\} - 1); \quad \varphi^a_- = \tilde{M}\tilde{\varphi}^a_-$$
$$\varphi^a_+ = \tilde{M}^{-1}\tilde{\varphi}^a_+; \quad \varphi^0_+ = \tilde{M}^{-1}\tilde{\varphi}^0_+$$
$$e = \tilde{M}^{-1}\tilde{e}; \quad b = \tilde{M}\tilde{b} \tag{18}$$

where

$$\tilde{M} = 1 + \frac{g}{2m}\varphi^0_- = \exp\{\frac{gh}{2m}\} \tag{19}$$

At the surface $\varphi^a_- = 0$, the equation $(\tilde{\varphi}^{\Omega}_-)^a = 0$, implies $\eta^a = 0$.

No ambiguity!

The divergency index of a diagram with L_Φ external lines of the field Φ:

$$n = 4 - 2L_{\varphi^0_+} - 2L_{\varphi^a_+} - L_A - L_e - L_b - L_h - L_B - L_\sigma \qquad (20)$$

All the diagrams with more than four external lines are convergent.

The model is explicitly renormalizable!

Unitarity.

The model includes many unphysical (ghost) fields: $\varphi_+^\alpha, (\alpha = 0, 1, 2, 3,), h$, $\varphi^a(B^a)(a = 1, 2, 3), e^\alpha, b^\alpha, A_0^a$. The unitarity in the physical subspace, including only A_i^a, σ should be proven.

The Lagrangian L was invariant with respect to the supersymmetry transformaions:

$$\begin{aligned} \delta\varphi_-^a &= -b^a \\ \delta\varphi_-^0 &= -b^0 \\ \delta e^a &= \varphi_+^a \\ \delta e^0 &= \varphi_+^0 \\ \delta b &= 0 \\ \delta\varphi_+^\alpha &= 0 \\ \alpha &= 0, 1, 2, 3. \end{aligned} \tag{21}$$

This invariance induces the corresponding symmetry transformations of the variables $\tilde{\varphi}^{\alpha}_{+}, h, \tilde{e}, \tilde{b}$,which leave invariant the Lagrangian $\tilde{L}$.

The asymptotic theory is invariant with respect to the supersymmetry transformations

$$\delta\tilde{\varphi}^a_- = 0; \delta A^a_\mu = m^{-1}\partial_\mu \tilde{b}^a; \delta h = -\tilde{b}^0; \delta\varphi^a = 0. \tag{22}$$

This invariance provides the conservation of the charge Q, and unitarity of the scattering matrix in the subspace of the states annihilated by Q_0: $Q_0|\psi >_{as} = 0$

Together with the gauge invariance it guarantees the unitarity of the S-matrix in the space including only physical states A^a_i, σ.

Conclusion

1.A unique covariant quantization of the Higgs-Kibble (Weinberg-Salam) model beyond the perturbation theory is possible.

The model is renormalizable in the ambiguity free Lorentz invariant gauge.

The necessary counterterms preserve the symmetries, which provide the unitarity of the renormalized theory and preserve the gauge invariance. However a redefinition of the parameters and the fields is needed.

The crucial role for all this consruction must be played by the nonperturbative calculations.